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PAN-ARCTIC OUTLOOK — Hamilton

1. Extent Projection

A Gompertz (asymmetrical S curve) model estimated by iterative least squares suggests a mean September 2011 ice extent of **4.4 million km²** (NSIDC). *The 95% confidence interval for this projection is approximately 3.5 to 5.3 million km².*

2. Methods / Techniques

Figure 1 shows this naive, purely statistical model. It predicts September mean extent simply from a Gompertz curve representing the trend over previous years. Estimation data are the NSIDC monthly mean extent reports from September 1979 through September 2010.

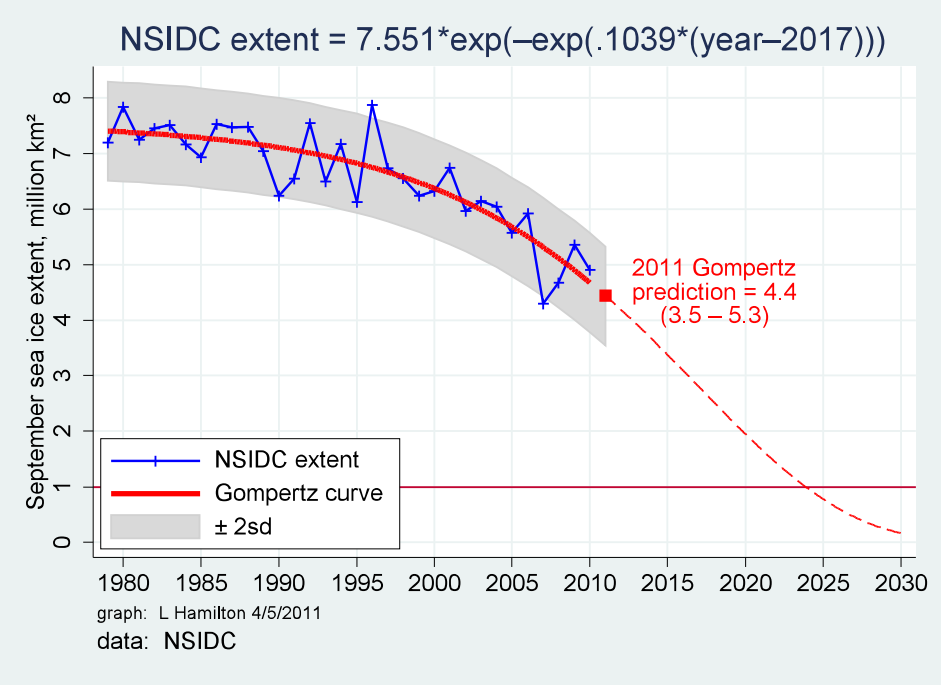

Figure 1

Parameters for the model are estimated via iterative least squares, using the **nl** procedure of Stata. Figure 1 also shows confidence bands calculated as the prediction plus or minus twice the standard deviation of the residuals.

In the command below, **gom3** specifies a 3-parameter Gompertz curve. **nsidc** refers to September mean NSIDC sea ice extent, in millions of km^2. **year** refers to the calendar year.

```
. nl gom3: nsidc year
(obs = 32)

      Source |       SS       df       MS                      
-------------+------------------------------                   Number of obs =        32
       Model |  1404.20492     3  468.068308                    R-squared     =    0.9956
    Residual |  6.14037236    29  .211736978                    Adj R-squared =    0.9952
-------------+------------------------------                   Root MSE      =  .4601489
       Total |  1410.34529    32  44.0732905                    Res. dev.     =  37.98485

3-parameter Gompertz function, nsidc = b1*exp(-exp(-b2*(year - b3)))
------------------------------------------------------------------------------
       nsidc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         /b1 |    7.55082   .2923261    25.83   0.000     6.952946    8.148694
         /b2 |  -.1039094   .0310938    -3.34   0.002    -.1675033   -.0403155
         /b3 |   2017.088   2.499096   807.13   0.000     2011.977    2022.199
------------------------------------------------------------------------------
```

The squared correlation between observed and predicted values (not shown) is $r^2 = .77$. There is no significant autocorrelation among the residuals, as tested by Ljung–Box *Q* statistics.

```
. predict resid, resid
. corrgram resid, lag(6)
                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1       -0.2199  -0.2215   1.6968  0.1927         -|                 -|
2       -0.0757  -0.1398   1.9048  0.3858          |                 -|
3       -0.2247  -0.3082    3.799  0.2840         -|                --|
4        0.0467  -0.1357   3.8837  0.4220          |                 -|
5        0.1589   0.1787   4.9013  0.4280          |-                 |-
6       -0.1763  -0.2768   6.2017  0.4010         -|                --|
```

3. Rationale

This is a naive model proposed at the start of the 2011 melt season. As more data become available between now and September, I expect that prediction can be improved.

Most trend-line analyses of Arctic sea ice have used linear, quadratic, exponential or logistic models. The Gompertz curve appears more plausible than any of these alternatives, in several respects.

- It allows for a gradually accelerating decline in the 1970s and 80s, going back even to earlier years.
- The decline later steepens at an accelerating rate, as observed since the mid-2000s.
- Model predictions do not cross or exactly reach zero extent. Rather they approach this limit asymptotically.

Although out-of-sample extrapolation is just speculative, it is interesting to note that the model suggests extent falling below 1 million km^2 before 2025.

For comparison, I fit similar Gompertz curve models to northern ice *area*, as seen in **Figure 2**. This suggests a September 2011 NSIDC area prediction of 3.1 million km^2 (confidence interval 2.4–3.8). As with the extent model in Figure 1, extrapolating the area model in Figure 2 implies a decline to very low values by 2025.

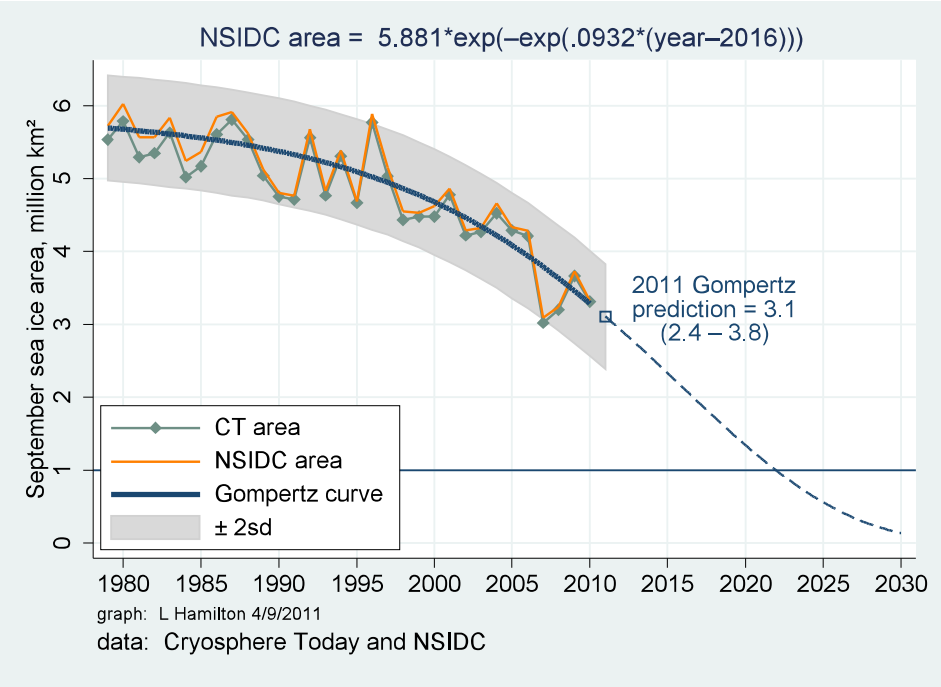

Figure 2

Finally, I fit a similar Gompertz model to PIOMAS estimates of Arctic ice *volume*. This yields a September 2011 volume prediction of 4.2 thousand km^3. Extrapolating, the volume curve falls to very low levels (below 2,000 km^3) sooner than the extent or area curves do.

Further elaboration and discussion of these simple models can be found in two Arctic Sea Ice Blog posts:

http://neven1.typepad.com/blog/2011/04/trends-in-arctic-sea-ice-extent.html

http://neven1.typepad.com/blog/2011/04/trends-in-arctic-sea-ice-volume.html

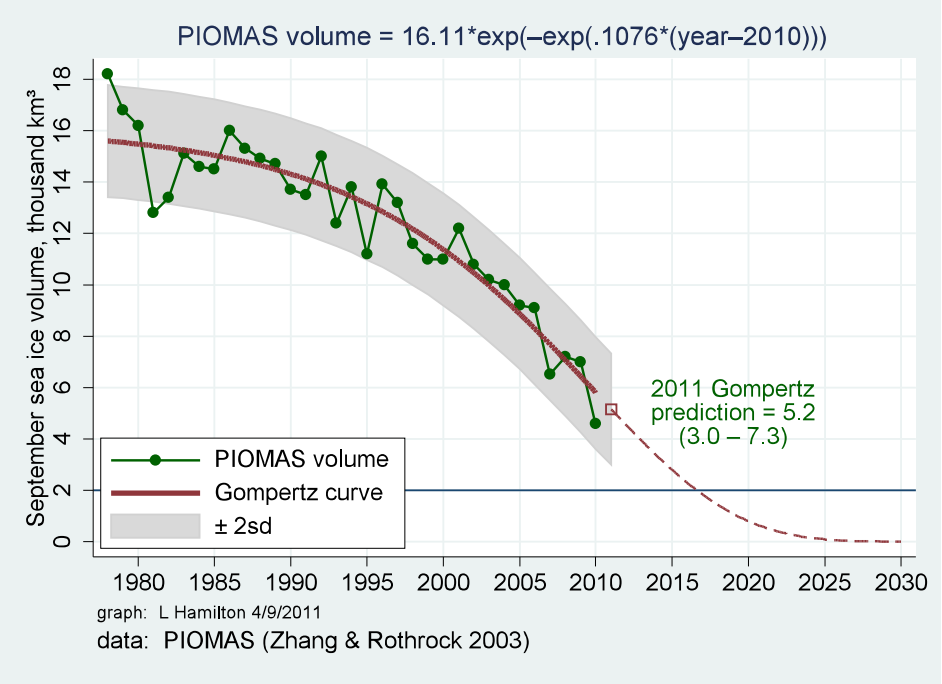

Figure 3

4. Executive Summary

Mean September NSIDC ice extent 1980–2010 appears reasonably well described by the naive but physically plausible Gompertz statistical model, graphed as an asymmetrical S-shaped curve. **This model yields a predicted September 2011 extent of 4.4 million km^2**, with a confidence interval from 3.5 to 5.3 million km^2. Refinements to this predict should be possible when we have further information, as the 2011 melt season progresses.

5. Estimate of Forecast Skill

Over 1979–2010, the standard deviation of NSIDC September ice extent is 0.92 million km^2. The standard deviation of residuals from the model in Figure 1 is just 0.45 million km^2. Thus, the model explains about 77% of the variance (Var[*predicted*]/Var[*observed*]) = .77) in observed September ice extent. Using only 1979–2009 data, a similar model predicts the 2010 extent as 4.5 (3.7–5.4); the actual 2010 value was 4.9, well within this range.

Gray bands show a range of plus or minus two standard deviations around the curves in Figures 1–3. Regarding predicted 2011 extent (4.4 million km^2 as seen in Figure 1), these suggest a confidence interval from 3.5 to 5.3 million km^2.