

PAN-ARCTIC OUTLOOK – Hamilton

1. Extent Projection

A Gompertz (asymmetrical S curve) model estimated by iterative least squares suggests a mean September 2012 ice extent of **4.3 million km²** (NSIDC). *The 95% confidence interval for this prediction ranges from 3.4 to 5.1 million km².*

2. Methods / Techniques

Figure 1 shows this naive, purely statistical model. It predicts September mean extent from a Gompertz curve representing the trend over previous years. Estimation data are the NSIDC monthly mean extent reports from September 1979 through September 2011.

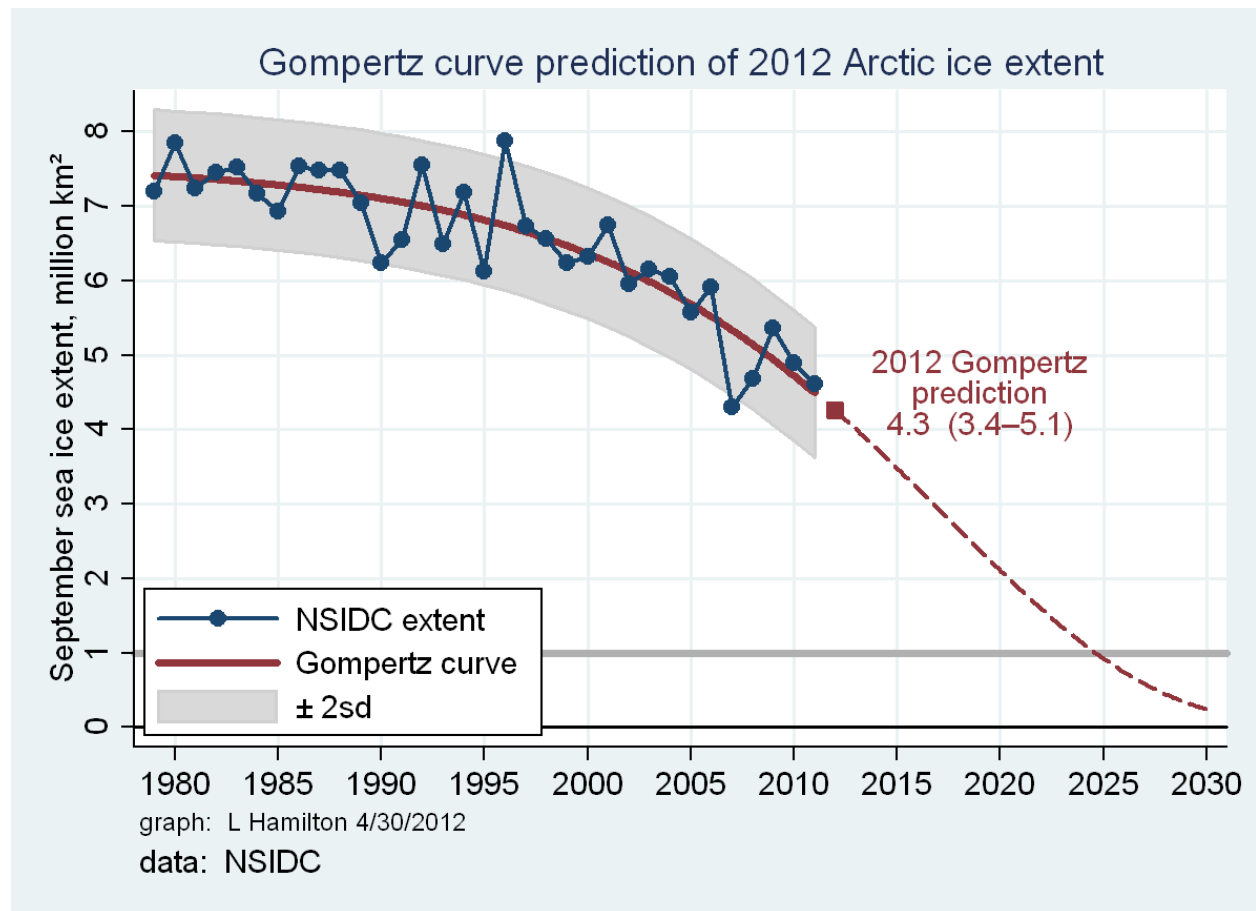


Figure 1

Parameters for the model are estimated via iterative least squares, using the **nl** procedure of Stata. Figure 1 also shows confidence bands calculated as the prediction plus or minus twice the standard deviation of the residuals.

In the command below, **gom3** specifies a 3-parameter Gompertz curve. **extent** refers to September mean NSIDC sea ice extent, in millions of km². **year** refers to the calendar year.

```
. use C:\data\Arctic9.dta, clear
(Arctic September mean sea ice 1979-2011)
```

```
. nl gom3: extent year, nolog
(obs = 33)
```

Source	SS	df	MS		
Model	1425.43798	3	475.145994	Number of obs =	33
Residual	6.15941312	30	.205313771	R-squared =	0.9957
				Adj R-squared =	0.9953
				Root MSE =	.4531156
Total	1431.5974	33	43.3817393	Res. dev. =	38.25858

3-parameter Gompertz function, extent = b1*exp(-exp(-b2*(year - b3)))

extent	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
/b1	7.580278	.291652	25.99	0.000	6.984645	8.175911
/b2	-.0995915	.0271646	-3.67	0.001	-.155069	-.044114
/b3	2017.531	2.173212	928.36	0.000	2013.093	2021.969

The squared correlation between observed and predicted values (not shown) is $r^2 = .79$. There is no significant autocorrelation among the residuals, as tested by Ljung-Box Q statistics.

```
. predict resid, resid
. corrgram resid, lag(6)
```

LAG	AC	PAC	Q	Prob>Q	-1	0	1	-1	0	1
					[Autocorrelation]			[Partial Autocor]		
1	-0.2135	-0.2138	1.6449	0.1996	-			-		
2	-0.0599	-0.1094	1.7787	0.4109						
3	-0.2277	-0.3000	3.7748	0.2868	-			--		
4	0.0233	-0.1280	3.7963	0.4343				-		
5	0.1641	0.1840	4.9073	0.4273	-			-		
6	-0.1815	-0.2761	6.3169	0.3886	-			--		

3. Rationale

This is a naive model proposed at the start of the 2012 melt season. Most trend-line analyses of Arctic sea ice have used linear, quadratic, exponential or logistic models. The Gompertz curve appears preferable to these alternatives in several respects.

- It follows the observed pattern of gradually accelerating decline in the 1970s and 80s.
- The decline later steepens at an accelerating rate, as observed since the mid-2000s.
- Model predictions do not cross or exactly reach zero extent. Rather they approach this limit asymptotically.
- The asymmetrical-S shape bears a qualitative resemblance to results from much more elaborate physical models, such as those reported by the IPCC (2007).

Although out-of-sample extrapolation of this non-physical model is purely speculative, it is interesting to note the suggestion of extent falling below 1 million km² by 2025.

If we add Gaussian noise with the same standard deviation as past residuals to the projected future curve, we see behavior like the four examples in Figure 2.

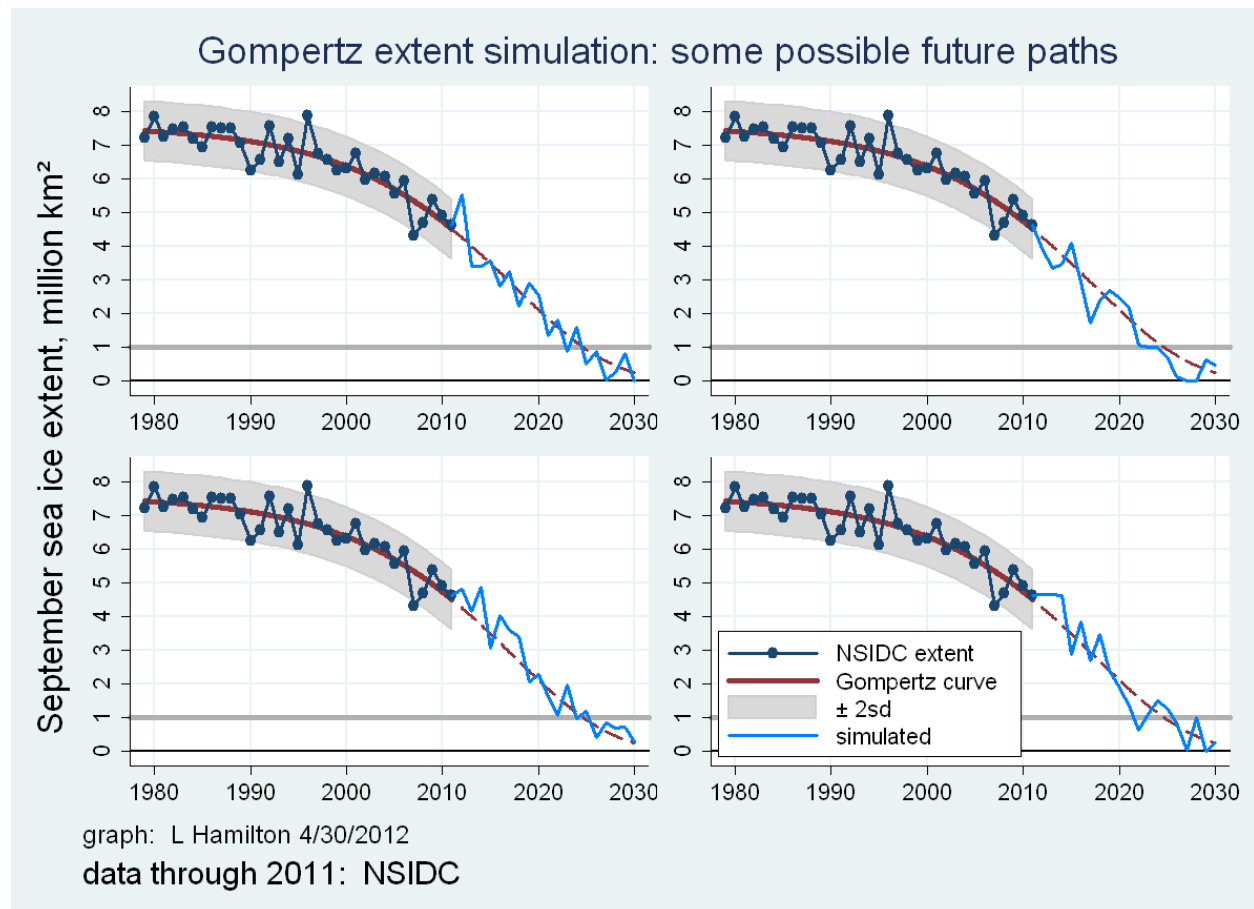


Figure 2

5. Estimate of Forecast Skill

Over 1979–2011, the standard deviation of NSIDC September ice extent is 0.97 million km². The standard deviation of residuals from the model in Figure 1 is just 0.44 million km². The model explains about 79% of the variance ($\text{Var}[\textit{predicted}]/\text{Var}[\textit{observed}] = .79$) in observed September ice extent. Using only 1979–2010 data, a similar model predicted the 2011 extent as 4.4 (3.5–5.3); the observed 2011 value was 4.6, well within this range.

Gray bands show a range of plus or minus two standard deviations around the curves in Figures 1 and 2. That suggests a confidence interval from 3.4 to 5.1 million km² for the 2012 prediction.

Similar Gompertz models, applied to September 1979–2010 data (or with the longer Uni Bremen time series, 1972–2010) produced the following predictions for mean September 2011 extent, area and volume:

	Predicted	Observed	
	2011	2011	units
NSIDC extent	4.4	4.6	million km ²
Uni Bremen extent	4.6	4.6	million km ²
NSIDC area	3.1	3.2	million km ²
PIOMAS volume	4.9	4.2	thousand km ³