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PAN-ARCTIC OUTLOOK — Hamilton

1. Extent Projection

A Gompertz (asymmetrical S curve) model estimated by iterative least squares, looking one year ahead, suggests a mean September 2013 ice extent of **3.8 million km^2** (NSIDC). *The 95% confidence interval for this prediction ranges from 2.9 to 4.7 million km^2.*

2. Methods / Techniques

Figure 1 shows this naive, purely statistical model. It predicts September mean extent from a Gompertz curve representing the trend over previous years. Estimation data are the NSIDC monthly mean extent reports from September 1979 through September 2012. Thus, the September 2013 extent prediction was calculated in October 2012, almost one year in advance.

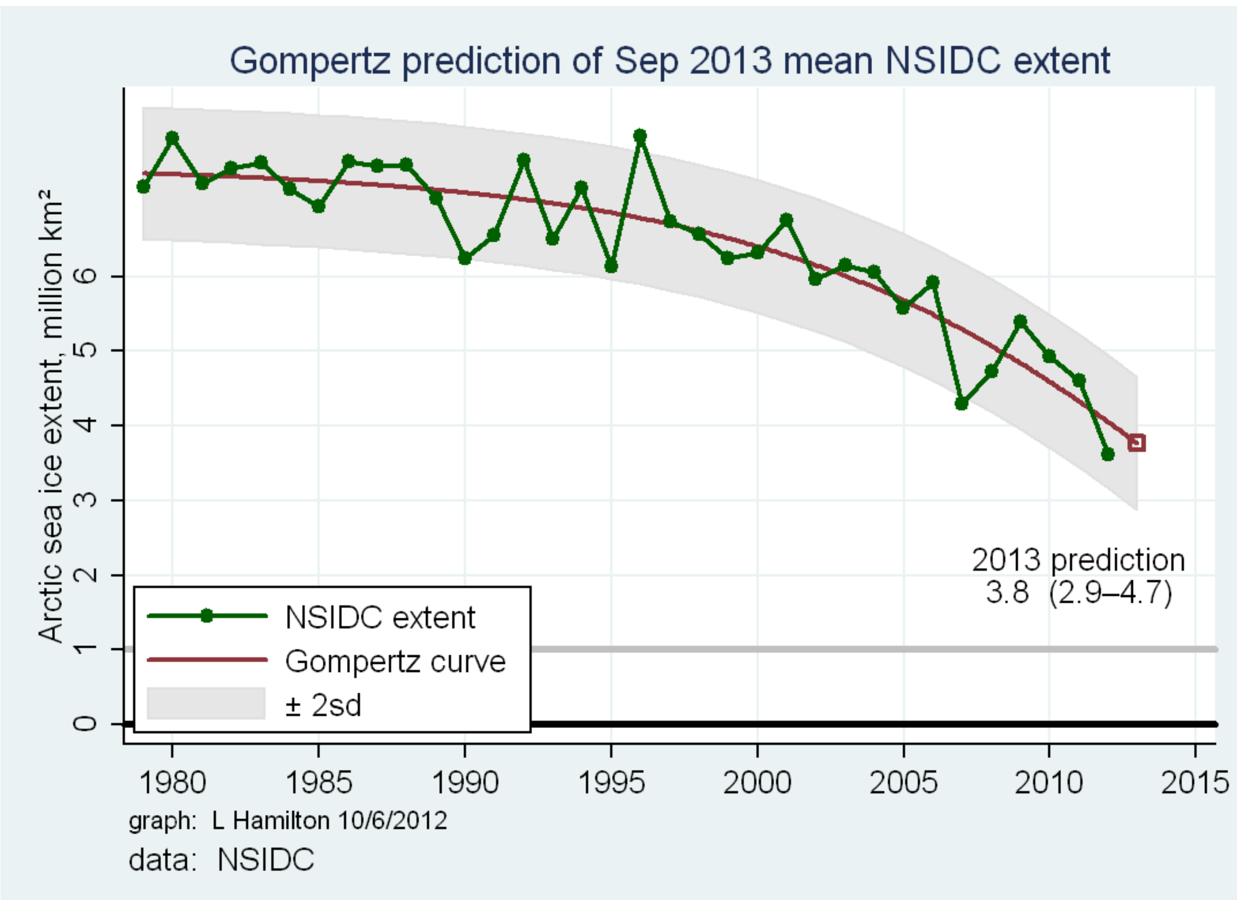

Figure 1

Parameters for the model are estimated via iterative least squares, using the **nl** procedure of Stata. Figure 1 also shows confidence bands calculated as the prediction plus or minus twice the standard deviation of the residuals.

In the command below, **gom3** specifies a 3-parameter Gompertz curve. ***extent*** refers to September mean NSIDC sea ice extent, in millions of km^2. ***year*** refers to the calendar year.

. nl gom3: *extent year*, nolog

```
 (obs = 34)

      Source |       SS       df       MS
-------------+------------------------------         Number of obs =        34
       Model |  1439.26373     3  479.754577         R-squared     =    0.9955
    Residual |  6.45366068    31  .208182602         Adj R-squared =    0.9951
-------------+------------------------------         Root MSE      =  .4562703
       Total |  1445.71739    34  42.5210998         Res. dev.     =  39.98958

3-parameter Gompertz function, extent = b1*exp(-exp(-b2*(year - b3)))
------------------------------------------------------------------------------
      extent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         /b1 |   7.484117    .228622    32.74   0.000     7.017839    7.950394
         /b2 |  -.1139896   .0248898    -4.58   0.000    -.1647526   -.0632265
         /b3 |   2016.297   1.474089  1367.83   0.000     2013.291    2019.304
------------------------------------------------------------------------------
```

The squared correlation between observed and predicted values is $r^2 = .83$. There is no significant autocorrelation among the residuals, as tested by Ljung–Box *Q* statistics.

. predict *resid*, resid
. corrgram *resid*, lag(6)

```
                                          -1       0       1 -1       0       1
LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1      -0.2150  -0.2224  1.7148  0.1904          -|                 -|
2      -0.0833  -0.1352  1.9805  0.3715           |                 -|
3      -0.2768  -0.3622  5.0052  0.1714         --|                --|
4       0.0362  -0.1899  5.0585  0.2813           |                 -|
5       0.2416   0.1606   7.522  0.1846           |-                 |-
6      -0.1963  -0.2710  9.2069  0.1623          -|                --|
```

The same approach, using Gompertz curves fit to 1979–2012 data, also suggests year-in-advance predictions for mean September 2013 sea ice area or volume, or for the one-day minimum area (Cryosphere Today data). **Figure 2** graphs all four predictions, with confidence intervals.

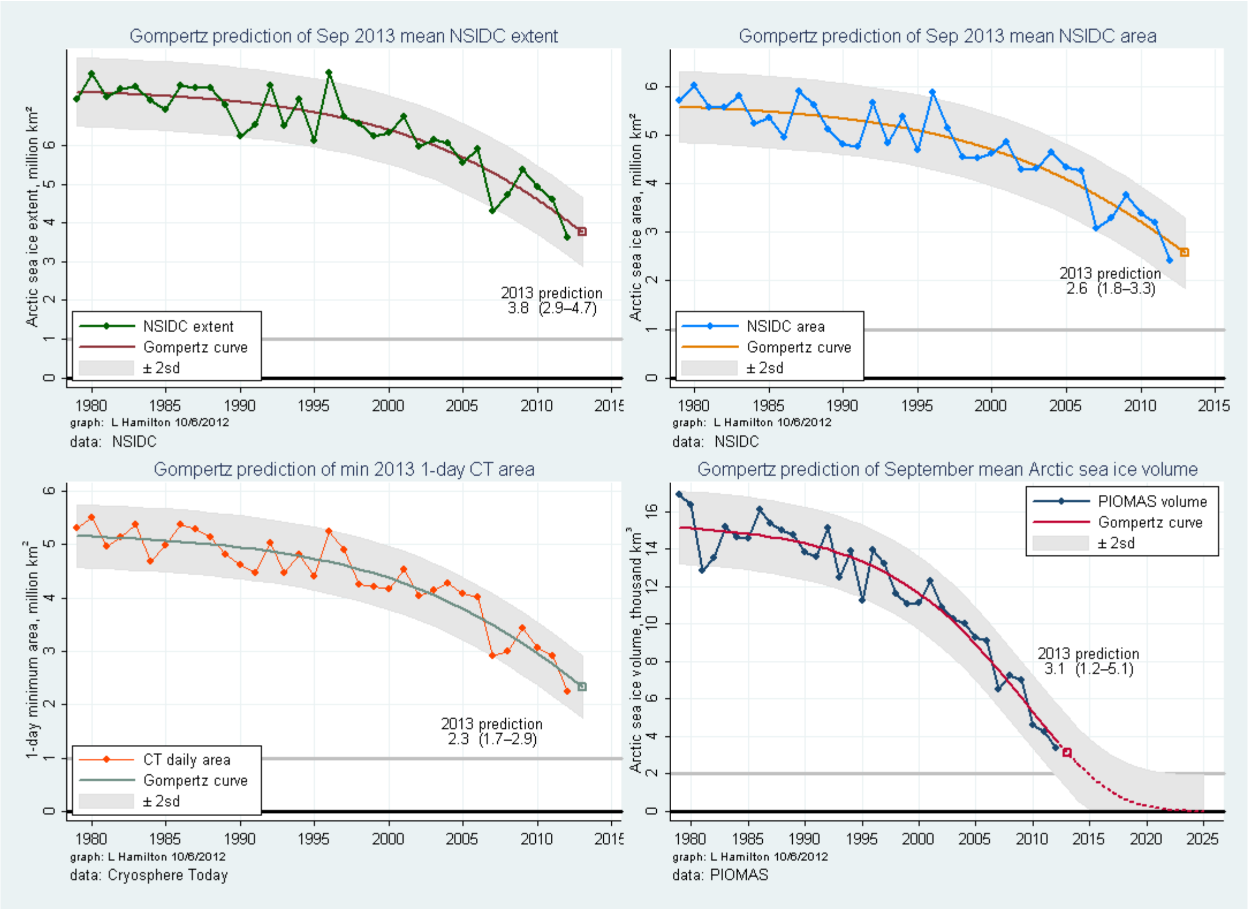

Figure 2

Horizontal lines in Figure 2 depict "virtually ice-free" conditions, considered to be September area or extent below 1 million square kilometers, or volume below 2,000 cubic kilometers. The volume curve is approaching ice-free conditions more quickly than area or extent, which implies that the remaining surface increasingly consists of thin ice — which has the potential for very rapid nonlinear declines in the future.

3. Rationale

These are naive models proposed in October 2012, at the end of the 2012 melt season. Most trend-line analyses of Arctic sea ice have used linear, quadratic, exponential or logistic models. The Gompertz curve appears preferable to these alternatives in several respects.

- It follows the observed pattern of gradually accelerating decline in the 1970s and 80s.
- The decline later steepens at an accelerating rate, as observed since the mid-2000s.
- Model predictions do not cross or exactly reach the physical extent limit of zero. Rather they approach this limit asymptotically.
- The asymmetrical-S shape bears a qualitative resemblance to results from much more elaborate physical models, such as those reported by the IPCC (2007).

Although out-of-sample extrapolation of this non-physical model is purely speculative, it is interesting to note the suggestion of extent falling below 1 million km^2 by 2025, and volume below 2,000 km^3 well before 2020.

If we add Gaussian noise with the same standard deviation as past residuals to the projected future curve, we see behavior like the four examples in **Figure 3**, which are based on 1979–2011 data.

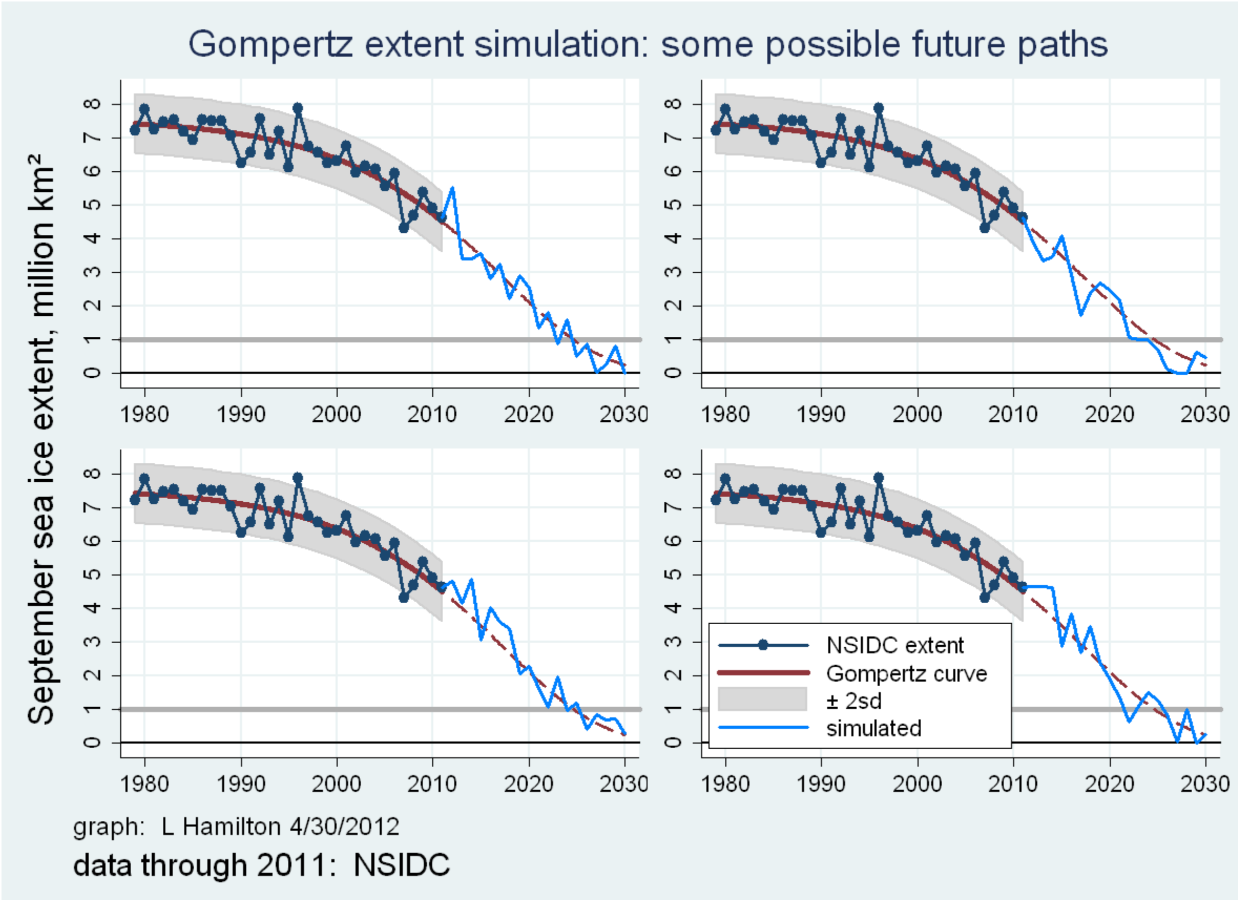
Gompertz extent simulation: some possible future paths
September sea ice extent, million km²
NSIDC extent
Gompertz curve
± 2sd
simulated
graph: L Hamilton 4/30/2012
data through 2011: NSIDC

Figure 3

5. Estimate of Forecast Skill

Gray bands show a range of plus or minus two standard deviations around the curves in Figures 1–3. In the case of mean September 2013 extent, that suggests a confidence interval from 2.9 to 4.7 million km^2.

Over 1979–2012, the standard deviation of NSIDC September ice extent is 1.07 million km^2. The standard deviation of residuals from the model in Figure 1 is just 0.44 million km^2. The model explains about 83% of the variance (Var[*predicted*]/Var[*observed*]) = .83) in observed September ice extent.

Similar Gompertz models, applied to September 1979–2010 data (or with the longer University of Bremen time series, 1972–2010) produced the following predictions for mean September 2011 extent, area and volume:

	Predicted 2011	**Observed 2011**	**units**
NSIDC extent	4.4	4.6	million km^2
Uni Bremen extent	4.6	4.6	million km^2
NSIDC area	3.1	3.2	million km^2
PIOMAS volume	4.9	4.2	thousand km^3

Applied to September 1979–2011 data, the same Gompertz approach gave these predictions for 2012. In each case the observed values turned out to be lower than predicted.

	Predicted 2012	**Observed 2012**	**units**
NSIDC extent	4.3	3.6	million km^2
NSIDC area	3.0	2.4	million km^2
CT area 1-day	2.7	2.2	million km^2
PIOMAS volume	4.0	3.4	thousand km^3

The year-in-advance nature of these predictions should be emphasized. They are most directly comparable to other year-in-advance methods. These Gompertz models could also be viewed as naïve null hypotheses for comparison with more sophisticated models that incorporate physics and observations made later in the season.