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PAN-ARCTIC OUTLOOK

1. Contributor name
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2. Executive summary
A Gompertz (asymmetric S curve) model estimated by iterative least squares, looking one year ahead, suggests a mean September 2015 ice extent of **4.2 million km^2**. Past variations suggest a 95% confidence interval for this prediction ranging from 3.2 to 5.2 million km^2 (±1.0).

3. Type
statistical

4. September monthly average projection
4.2 million km^2

5. Explanation of method
This is a naive, purely statistical model — a null hypothesis, in effect. It predicts September mean extent from a Gompertz curve representing the trend over previous years. Estimation data are the NSIDC monthly mean extent reports from September 1979 through September 2014. Thus, the September 2015 extent prediction is calculated from data available in October 2014, one year in advance.

Parameters for the model are estimated via iterative least squares using the **nl** procedure of Stata (Hamilton 2013).

6. Uncertainty
Past variations suggest a 95% confidence interval for this prediction ranging from 3.2 to 5.2 million km^2 (±1.0).

7. Explanation of uncertainty
Over 1979 to 2014 the standard deviation of residuals from this model is about 0.5 million km^2. The uncertainty suggested is plus or minus two standard deviations, or ±1.0 million km^2.

More details are given on the following pages.

8. Extent Projection

A Gompertz (asymmetric S curve) model estimated by iterative least squares, looking one year ahead, suggests a mean September 2015 ice extent of **4.2 million km^2**. Past variations suggest a 95% confidence interval for this prediction ranging from 3.2 to 5.2 million km^2 (±1.0).

9. Methods / Techniques

Figure 1 shows the naive, purely statistical model. It predicts September mean extent from a Gompertz curve representing the trend over previous years. Estimation data are the NSIDC monthly mean extent reports from September 1979 through September 2014. Thus, the September 2015 extent prediction is calculated from data available in October 2014, one year in advance.

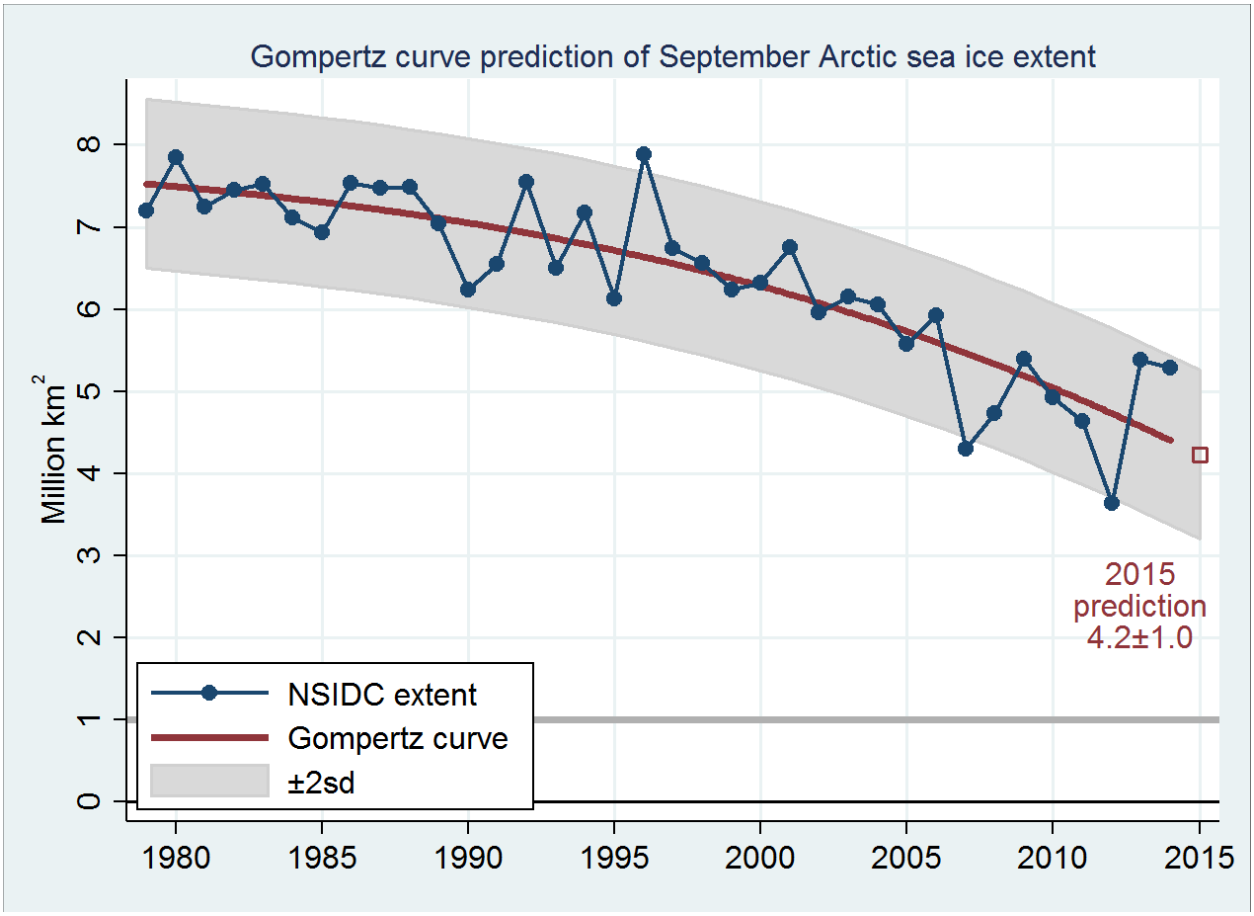

Figure 1

Parameters for the model are estimated via iterative least squares using the **nl** procedure of Stata (Hamilton 2013). Figure 1 also shows confidence bands calculated as the prediction plus or minus twice the standard deviation of the residuals.

In the command below, **gom3** specifies a 3-parameter Gompertz curve. ***extent*** refers to September mean NSIDC sea ice extent, in millions of km^2. ***year*** refers to the calendar year.

```
. nl gom3: extent year, nolog
(obs = 36)

      Source |       SS             df       MS
-------------+------------------------------    Number of obs =         36
       Model |  1487.8384            3   495.94612    R-squared     =     0.9938
    Residual |  9.2443303           33  .280131222    Adj R-squared =     0.9933
-------------+------------------------------    Root MSE      =   .5292742
       Total |  1497.0827           36  41.5856303    Res. dev.     =   53.22127

3-parameter Gompertz function, extent = b1*exp(-exp(-b2*(year - b3)))
------------------------------------------------------------------------------
      extent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         /b1 |   8.002348   .6210486    12.89   0.000     6.738815    9.265881
         /b2 |  -.0645248   .0231513    -2.79   0.009    -.1116264   -.0174232
         /b3 |   2021.892   2.643788   764.77   0.000     2016.513    2027.271
------------------------------------------------------------------------------
```

In this model the first parameter, b1 = 8.00, gives the asymptotic starting level, 8.00 million km^2. The third parameter, b3 = 2021.89, gives the inflection point where this curve shifts from a steepening rate of decline to a slowing rate of decline: during the year 2022. The second parameter, b2 = –.06, controls the rate of change in the decline.

There is no significant autocorrelation ($p > .10$) among the residuals, as tested by Ljung–Box Q statistics.

```
. predict resid, resid
. corrgram resid, lag(6)
                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1       -0.0885  -0.0961   .30619  0.5800          |                  |
2       -0.1116  -0.1564   .80693  0.6680          |                 -|
3       -0.1524  -0.2330   1.7692  0.6217         -|                 -|
4        0.1327   0.1451   2.5217  0.6408          |-                 |-
5        0.2090   0.3303   4.4499  0.4866          |-                 |--
6       -0.3180  -0.4475   9.0626  0.1701        --|              ---|
```

10. Rationale

This naive, curvilinear-trend model is based on data through the end of the 2014 melt season. Most trend-line analyses of Arctic sea ice have used linear, quadratic, exponential or logistic models. The Gompertz curve appears preferable to these alternatives in several respects.

- It follows the observed pattern of gradually accelerating decline in the 1970s and 80s.
- The decline later steepens at an accelerating rate, as observed since the mid-2000s.
- The asymmetrical-S shape bears a qualitative resemblance to results from much more elaborate physical models, such as those reported by the IPCC (2007).
- Extrapolated (as speculation) into the future, model predictions do not fall below zero extent. Rather they approach this physical limit asymptotically.

A recent meta-analysis of the ensemble skill of 309 contributions to the SEARCH Sea Ice Outlook over 2008–2013 (Stroeve et al. 2014) found that they show collective skill (median prediction near the true extent value) in years when sea ice extent falls close to its long-term downward trend. They collectively fail (median prediction distant from the true extent) in years when sea ice extent substantially departs from this trend. The Gompertz trend-based prediction given here is therefore proposed as a rough null hypothesis: what we would expect if there is nothing but a continuation of the past 36 years' curvilinear trend.

11. Estimate of Forecast Skill

Gray bands in Figure 1 show a range of plus or minus two standard deviations around the curve. That suggests a confidence interval from 3.2 to 5.2 million km^2 for the September 2015 extent prediction.

Over 1979–2014, the standard deviation of NSIDC September ice extent is 1.07 million km^2. The standard deviation of residuals from the model in Figure 1 is just 0.51 million km^2. The squared correlation between observed and predicted values is $r^2 = .77$.

Similar Gompertz models estimated from data through the previous year suggest the following predictions for mean September extent in 2011 to 2014.

	Predicted	**Observed**
2011	4.5	4.6
2012	4.3	3.6
2013	3.8	5.3
2014	4.1	5.3
2015	4.2	—

References

Hamilton, L.C. 2013. *Statistics with Stata*, version 12. Belmont, CA: Cengage

Stroeve, J., L.C. Hamilton, C.M. Bitz, E. Blanchard-Wrigglesworth. 2014. "Predicting September sea ice: Ensemble skill of the SEARCH Sea Ice Outlook." *Geophysical Research Letters* 41:2411–2418. doi: 10.1002/2014GL059388